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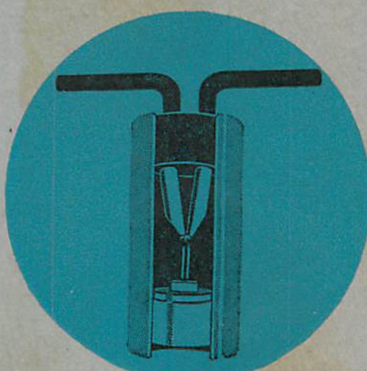

by

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Presented at

The 287th meeting of the

AMERICAN PHYSICAL SOCIETY

Chicago, Illinois, November 27, 1948

Published in

THE PHYSICAL REVIEW

Vol. 75, pp. 1208-1225, April 15, 1949

Printed in the UNITED STATES OF AMERICA

Physical Principles Involved in Transistor Action

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(Received December 27, 1948)

The transistor in the form described herein consists of two point-contact electrodes, called emitter and collector, placed in close proximity on the upper face of a small block of germanium. The base electrode, the third element of the triode, is a large area, low resistance contact on the lower face. Each point contact has characteristics similar to those of the high back-voltage rectifier. When suitable d.c. bias potentials are applied, the device may be used to amplify a.c. signals. A signal introduced between the emitter and base appears in amplified form between collector and base. The emitter is biased in the positive direction, which is that of easy flow. A larger negative or reverse voltage is applied to the collector. Transistor action depends on the fact that electrons in semiconductors can carry current in two different ways: by excess or conduction electrons and by defect "electrons" or holes. The germanium used is n-type, i.e., the carriers are conduction electrons. Current from the emitter is composed in large part of holes, i.e., of carriers of opposite sign to those normally in excess in the body of the block. The holes are attracted by the field of the collector current, so that a large part of the emitter current, introduced at low impedance, flows into the collector circuit and through a high impedance load. There is a voltage gain and a power gain of an input signal. There may be current amplification as well.

The influence of the emitter current, I_e, on collector current, I_c, is expressed in terms of a current multiplication factor, α, which gives the rate of change of I_c with respect to I_e at constant collector voltage. Values of α in typical units range from about 1 to 3. It is shown in a general way how α depends on bias voltages, frequency, temperature, and electrode spacing. There is an influence of collector current on emitter current in the nature of a positive feedback which under some operating conditions may lead to instability.

The way the concentrations and mobilities of electrons and holes in germanium depend on impurities and on temperature is described briefly. The theory of germanium point contact rectifiers is discussed in terms of the Mott-Schottky theory. The barrier layer is such as to raise the levels of the filled band to a position close to the Fermi level at the surface, giving an inversion layer of p-type or defect conductivity. There is considerable evidence that the barrier layer is intrinsic and occurs at the free surface, independent of a metal contact. Potential probe tests on some surfaces indicate considerable surface conductivity which is attributed to the p-type layer. All surfaces tested show an excess conductivity in the vicinity of the point contact which increases with forward current and is attributed to a flow of holes into the body of the germanium, the space charge of the holes being compensated by electrons. It is shown why such a flow is to be expected for the type of barrier layer which exists in germanium, and that this flow accounts for the large currents observed in the forward direction. In the transistor, holes may flow from the emitter to the collector either in the surface layer or through the body of the germanium. Estimates are made of the field produced by the collector current, of the transit time for holes, of the space charge produced by holes flowing into the collector, and of the feedback resistance which gives the influence of collector current on emitter current. These calculations confirm the general picture given of transistor action.

I. INTRODUCTION

THE transistor, a semiconductor triode which in its present form uses a small block of germanium as the basic element, has been described briefly in the Letters to the Editor columns of the Physical Review.[1] Accompanying this letter were two further communications on related subjects.[2,3] Since these initial publications a number of talks describing the characteristics of the device and the theory of its operation have been given by the authors and by other members of the Bell Telephone Laboratories staff.[4] Several articles have appeared in the technical literature.[5] We plan to give here an outline of the history of the development, to give some further data on the characteristics and to discuss the physical principles involved. Included is a review of the nature of electrical conduction in germanium and of the theory of the germanium point-contact rectifier.

A schematic diagram of one form of transistor is shown in Fig. 1. Two point contacts, similar to those used in point-contact rectifiers, are placed in close proximity ($\sim$0.005–0.025 cm) on the upper surface of a small block of germanium. One of these, biased in the forward direction, is called the emitter. The second, biased in the reverse direction, is called the collector. A large area, low resistance contact on the lower surface, called the base electrode, is the third element of the triode. A physical embodiment of the device, as designed in large part by W. G. Pfann, is shown in Fig. 2. The transistor can be used for many functions now performed by vacuum tubes.

During the war, a large amount of research on

[1] J. Bardeen and W. H. Brattain, Phys. Rev. **74**, 230 (1948).
[2] W. H. Brattain and J. Bardeen, Phys. Rev. **74**, 231 (1948).
[3] W. Shockley and G. L. Pearson, Phys. Rev. **74**, 232 (1948).
[4] This paper was presented in part at the Chicago meeting of the American Physical Society, Nov. 26, 27, 1948. W. Shockley and the authors presented a paper on "The Electronic Theory of the Transistor" at the Berkeley meeting of the National Academy of Sciences, Nov. 15–17, 1948. A talk was given by one of the authors (W.H.B.) at the National Electronics Conference at Chicago, Nov. 4, 1948. A number of talks have been given at local meetings by J. A. Becker and other members of the Bell Telephone Laboratories Staff, as well as by the authors.
[5] Properties and characteristics of the transistor are given by J. A. Becker and J. N. Shive in Elec. Eng. **68**, 215 (1949). A coaxial form of transistor is described by W. E. Kock and R. L. Wallace, Jr. in Elec. Eng. **68**, 222 (1949). See also "The Transistor, A Crystal Triode," D. G. F. and F. H. R., Electronics, September (1948) and a series of articles by S. Young White in Audio Eng., August through December (1948).

the properties of germanium and silicon was carried out by a number of university, government, and industrial laboratories in connection with the development of point-contact rectifiers for radar. This work is summarized in the book of Torrey and Whitmer.[6] The properties of germanium as a semiconductor and as a rectifier have been investigated by a group working under the direction of K. Lark-Horovitz at Purdue University. Work at the Bell Laboratories[7] was initiated by R. S. Ohl before the war in connection with the development of silicon rectifiers for use as detectors at microwave frequencies. Research and development on both germanium and silicon rectifiers during and since the war has been done in large part by a group under J. H. Scaff. The background of information obtained in these various investigations has been invaluable.

The general research program leading to the transistor was initiated and directed by W. Shockley. Work on germanium and silicon was emphasized because they are simpler to understand than most other semiconductors. One of the investigations undertaken was the study of the modulation of conductance of a thin film of semiconductor by an electric field applied by an electrode insulated from the film.[3] If, for example, the film is made one plate of a parallel plate condenser, a charge is induced on the surface. If the individual charges which make up the induced charge are mobile, the conductance of the film will depend on the voltage applied to the condenser. The first experiments performed to measure this effect indicated that most of the induced charge was not mobile. This result, taken along with other unexplained phenomena such as the small contact potential difference between *n*- and *p*-type silicon[8] and the independence of the rectifying properties of the point-contact rectifier on the work function of the metal point led one of the authors to an explanation in terms of surface states.[9] This work led to the concept that space-charge barrier layers may be present at the free surfaces of semiconductors such as germanium and silicon, independent of a metal contact. Two experiments immediately suggested were to measure the dependence of contact potential on impurity concentration[10] and to measure the change of contact potential on illuminating the surface with light.[11] Both of these experiments were successful and confirmed the theory. It was while studying the latter effect with a silicon surface immersed in a liquid that it was found that the density of surface charges and the field in the space-charge region could be varied by applying a potential across an electrolyte in contact with the silicon surface.[12] While studying the effect of field applied by an electrolyte on the current voltage characteristic of a high back-voltage germanium rectifier, the authors were led to the concept that a portion of the current was being carried by holes flowing near the surface. Upon replacing the electrolyte with a metal contact transistor action was discovered.

The germanium used in the transistor is an *n*-type or excess semiconductor with a resistivity of the order of 10 ohm cm and is the same as the material used in high back-voltage germanium rectifiers.[13] All of the material we have used was prepared by J. C. Scaff and H. C. Theuerer of the metallurgical group of the Laboratories.

While different metals may be used for the contact points, most work has been done with phosphor bronze points. The spring contacts are made with wire from 0.002 to 0.005″ in diameter. The ends are cut in the form of a wedge so that the two contacts can be placed close together. The actual contact area is probably no more than about 10^{-6} cm^2.

The treatment of the germanium surface is similar to that used in making high back-voltage rectifiers.[14] The surface is ground flat and then etched. In some cases special additional treatments such as anodizing the surface or oxidation at 500°C have been used. The oxide films formed in these processes wash off easily and contact is made to the germanium surface.

The circuit of Fig. 1 shows how the transistor may be used to amplify a small a.c. signal. The emitter is biased in the forward (positive) direction so that a small d.c. current, of the order of 1 ma, flows into the germanium block. The collector is biased in the reverse (negative) direction with a

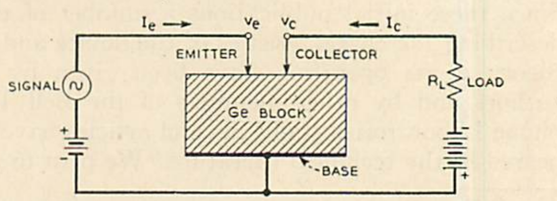

FIG. 1. Schematic of transistor showing circuit for amplification of an a.c. signal and conventional direction for currents. Note bias currents I_e and V_e are normally positive, I_c and V_c negative.

[6] H. C. Torrey and C. A. Whitmer, *Crystal Rectifiers* (McGraw-Hill Book Company, Inc., New York, 1948).
[7] J. H. Scaff and R. S. Ohl, Bell Sys. Tech. J. **26**, 1 (1947).
[8] Walter E. Meyerhof, Phys. Rev. **71**, 727 (1947).
[9] John Bardeen, Phys. Rev. **71**, 717 (1947).
[10] W. H. Brattain and W. Shockley, Phys. Rev. **72**, 345(L) (1947).
[11] Walter H. Brattain, Phys. Rev. **72**, 345(L) (1947).
[12] R. B. Gibney, formerly of Bell Telephone Laboratories, now at Los Alamos Scientific Laboratory, worked on chemical problems for the semiconductor group, and the authors are grateful to him for a number of valuable ideas and for considerable assistance.
[13] J. H. Scaff and H. C. Theuerer *Preparation of High Back Voltage Germanium Rectifiers* NDRC 14–155, Oct. 24, 1945. See reference 6, Chap. 12.
[14] The surface treatment is described in reference 6, p. 369.

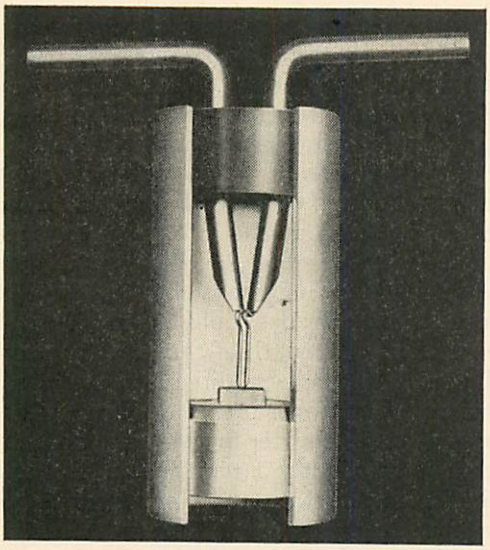

FIG. 2. Microphotograph of a cutaway model of a transistor.

higher voltage so that a d.c. current of a few milliamperes flows out through the collector point and through the load circuit. It is found that the current in the collector circuit is sensitive to and may be controlled by changes of current from the emitter. In fact, when the emitter current is varied by changing the emitter voltage, keeping the collector voltage constant, the change in collector current may be larger than the change in emitter current. As the emitter is biased in the direction of easy flow, a small a.c. voltage, and thus a small power input, is sufficient to vary the emitter current. The collector is biased in the direction of high resistance and may be matched to a high resistance load. The a.c. voltage and power in the load circuit are much larger than those in the input. An over-all power gain of a factor of 100 (or 20 db) can be obtained in favorable cases.

Terminal characteristics of an experimental transistor[15] are illustrated in Fig. 3, which shows how the current-voltage characteristic of the collector is changed by the current flowing from the emitter. Transistor characteristics, and the way they change with separation between the points, with temperature, and with frequency, are discussed in Section II.

The explanation of the action of the transistor depends on the nature of the current flowing from the emitter. It is well known that in semiconductors there are two ways by which the electrons can carry electricity which differ in the signs of the effective mobile charges.[16] The negative carriers are excess electrons which are free to move and are denoted by the term conduction electrons or simply electrons. They have energies in the conduction band of the crystal. The positive carriers are missing or defect "electrons" and are denoted by the term "holes." They represent unoccupied energy states in the uppermost normally filled band of the crystal. The conductivity is called n- or p-type depending on whether the mobile charges normally in excess in the material under equilibrium conditions are electrons (negative carriers) or holes (positive carriers). The germanium used in the transistor is n-type with about 5×10^{14} conduction electrons per cc; or about one electron per 10^8 atoms. Transistor action depends on the fact that the current from the emitter is composed in large part of *holes*; that is, of carriers of opposite sign to those normally in excess in the body of the semiconductor.

The collector is biased in the reverse, or negative direction. Current flowing in the germanium toward the collector point provides an electric field which is in such a direction as to attract the holes flowing from the emitter. When the emitter and collector are placed in close proximity, a large part of the hole current from the emitter will flow to the collector and into the collector circuit. The nature of the collector contact is such as to provide a high resistance barrier to the flow of electrons from the metal to the semiconductor, but there is little impediment to the flow of holes into the contact. This theory explains how the change in collector current might be as large as but not how it can be larger than the change in emitter current. The fact that the collector current may actually change more than the emitter current is believed to result from an alteration of the space charge in the barrier layer at the collector by the hole current flowing into the junction. The increase in density of space charge and in field strength make it easier for electrons to flow out from the collector, so that there is an increase in electron current. It is better to think of the hole current from the emitter as modifying the current-voltage characteristic of the collector, rather than as simply adding to the current flowing to the collector.

In Section III we discuss the nature of the conductivity of germanium, and in Section IV the theory of the current-voltage characteristic of a germanium-point contact. In the latter section we attempt to show why the emitter current is composed of carriers of opposite sign to those normally in excess in the body of germanium. Section V is concerned with some aspects of the theory of transistor action. A complete quantitative theory is not yet available.

There is evidence that the rectifying barrier in germanium is internal and occurs at the free

[15] The transistor whose characteristics are given in Fig. 3 is one of an experimented pilot production which is under the general direction of J. A. Morton.

[16] See, for example, A. H. Wilson, *Semi-Conductors and Metals* (Cambridge University Press, London, 1939) or F. Seitz, *The Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), Sec. 68.

surface, independent of the metal contact.[9,17] The barrier contains what Schottky and Spenke[18] call an inversion region; that is, a change of conductivity type. The outermost part of the barrier next to the surface is *p*-type. The *p*-type region is very thin, of the order of 10^{-5} cm in thickness. An important question is whether there is a sufficient density of holes in this region to provide appreciable lateral conductivity along the surface. Some evidence bearing on this point is described below.

Transistor action was first discovered on a germanium surface which was subjected to an anodic oxidation treatment in a glycol borate solution after it had been ground and etched in the usual way for diodes. Much of the early work was done on surfaces which were oxidized by heating in air. In both cases the oxide is washed off and plays no direct role. Some of these surfaces were tested for surface conductivity by potential probe tests. Surface conductivities, on a unit area basis, of the order of 0.0005 to 0.002 mhos were found.[2] The value of 0.0005 represents about the lower limit of detection possible by the method used. It is inferred that the observed surface conductivity is that of the *p*-type layer, although there has been no direct proof of this. In later work it was found that the oxidation treatment is not essential for transistor action. Good transistors can be made with surfaces prepared in the usual way for high back-voltage rectifiers provided that the collector point is electrically formed. Such surfaces exhibit no measurable surface conductivity.

One question that may be asked is whether the holes flow from the emitter to the collector mainly in the surface layer or whether they flow through the body of the germanium. The early experiments suggested flow along the surface. W. Shockley proposed a modified arrangement in which in effect the emitter and collector are on opposite sides of a thin slab, so that the holes flow directly across through the semiconductor. Independently, J. N. Shive made, by grinding and etching, a piece of germanium in the form of a thin flat wedge.[19] Point contacts were placed directly opposite each other on the two opposite faces where the thickness of the wedge was about 0.01 cm. A third large area contact was made to the base of the wedge. When the two points were connected as emitter and collector, and the collector was electrically formed, transistor action was obtained which was comparable to that found with the original arrangement. There is no doubt that in this case the holes are flowing directly through the *n*-type germanium from the emitter to the collector. With two points close together on a plane surface holes may flow either through the surface layer or through the body of the semiconductor.

Still later, at the suggestion of W. Shockley, J. R. Haynes[20] further established that holes flow into the body of the germanium. A block of germanium was made in the form of a thin slab and large area electrodes were placed at the two ends. Emitter and collector electrodes were placed at variable separations on one face of the slab. The field acting between these electrodes could be varied by passing currents along the length of the slab. The collector was biased in the reverse direction so that a small d.c. current was drawn into the collector. A signal introduced at the emitter in the form of a pulse was detected at a slightly later time in the collector circuit. From the way the time interval, of the order of a few microseconds, depends on the field, the mobility and sign of the carriers were determined. It was found that the carriers are positively charged, and that the mobility is the same as that of holes in bulk germanium (1000 cm^2/volt sec.).

These experiments clarify the nature of the excess conductivity observed in the forward direction in high back-voltage germanium rectifiers which has been investigated by R. Bray, K. Lark-Horovitz, and R. N. Smith[21] and by Bray.[22] These authors attributed the excess conductivity to the strong electric field which exists in the vicinity of the point contact. Bray has made direct experimental tests to observe the relation between conductivity and field strength. We believe that the excess conductivity arises from holes injected into the germanium at the contact. Holes are introduced because of the nature of the barrier layer rather than as a direct result of the electric field. This has been demonstrated by an experiment of E. J. Ryder and W. Shockley.[23] A thin slab of germanium was cut in the form of a pie-shaped wedge and electrodes placed at the narrow and wide boundaries of the wedge. When a current is passed between the electrodes, the field strength is large at the narrow end of the wedge and small near the opposite electrode. An excess conductivity was observed when the narrow end was made positive; none when the wide end was positive. The magnitude of the current flow was the same in both cases. Holes injected at the narrow end lower the resistivity in the region which contributes most to the over-all resistance. When the current is in the opposite direction, any holes injected enter in a region of low field and do not have sufficient lifetime to be drawn down to the narrow end and so do not alter the resistance very

[17] The nature of the barrier is discussed in Section IV.
[18] W. Schottky and E. Spenke, Wiss. Veroff. Siemens Werken, **18**, 225 (1939).
[19] John N. Shive, Phys. Rev. **75**, 689 (1949).
[20] J. R. Haynes and W. Shockley, Phys. Rev. **75**, 691 (1949).
[21] R. Bray, K. Lark-Horovitz, and R. N. Smith, Phys. Rev. **72**, 530 (1947).
[22] R. Bray, Phys. Rev. **74**, 1218 (1948).
[23] E. J. Ryder and W. Shockley, Phys. Rev. **75**, 310 (1949).

much. With some surface treatments, the excess conductivity resulting from hole injection may be enhanced by a surface conductivity as discussed above.

The experimental procedure used during the present investigation is of interest. Current voltage characteristics of a given point contact were displayed on a d.c. oscilloscope.[24] The change or modulation of this characteristic produced by a signal impressed on a neighboring electrode or point contact could be easily observed. Since the input impedance of the scope was 10 megohms, and the gain of the amplifiers such that the lower limit of sensitivity was of the order of a millivolt, the oscilloscope was also used as a very high impedance voltmeter for probe measurements. Means were included for matching the potential to be measured with an adjustable d.c. potential the value of which could be read on a meter. A micromanipulator designed by W. L. Bond was used to adjust the positions of the contact points.

II. SOME TRANSISTOR CHARACTERISTICS

The static characteristics of the transistor are completely specified by four variables which may be taken as the emitter and collector currents, I_e and I_c, and the corresponding voltages, V_e and V_c. As shown in the schematic diagram of Fig. 1, the conventional directions for current flow are taken as positive into the germanium and the terminal voltages are relative to the base electrode. Thus I_e and V_e are normally positive, I_c and V_c negative.

There is a functional relation between the four variables such that if two are specified the other two are determined. Any pair may be taken as the independent variables. As the transistor is essentially a current-operated device, it is more in accord with the physics involved to choose the currents rather than the voltages. All fields in the semiconductor outside of the space charge regions immediately surrounding the point contacts are determined by the currents, and it is the current flowing from the emitter which controls the current-voltage characteristic of the collector. The voltages are single-valued functions of the currents, but, because of inherent feedback, the currents may be double-valued functions of the voltages. In reference 1, the characteristics of an experimental transistor were shown by giving the constant voltage contours on a plot in which the independent variables I_e and I_c are plotted along the coordinate axes.

In the following we give further characteristics, and show in a general way how they depend on the spacing between the points, on the temperature, and on the frequency. The data were taken mainly on experimental set-ups on a laboratory bench, and are not to be taken as necessarily typical of the characteristics of finished units. They do indicate in a general way the type of results which can be obtained. Characteristics of units made in pilot production have been given elsewhere.[5]

The data plotted in reference 1 were taken on a transistor made with phosphor bronze points on a

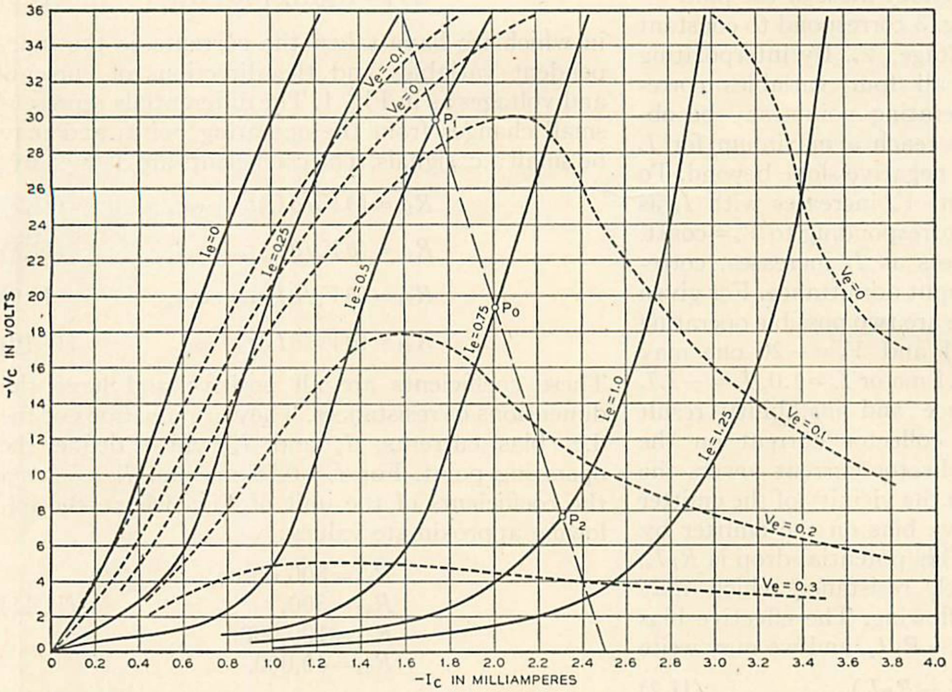

FIG. 3. Characteristics of an experimental transistor (see reference 15). The conventional directions for current and voltage are as in Fig. 1.

[24] This instrument was designed and built by H. R. Moore, who aided the authors a great deal in connection with instrumentation and circuit problems.

surface which was oxidized and on which potential probe tests gave evidence for considerable surface conductivity. The collector resistance is small in units prepared in this way. In Fig. 3 are shown the characteristics of a unit[15] in which the surface was prepared in a different manner. The surface was ground and etched in the usual way,[14] but was not subjected to the oxidation treatment. Phosphor bronze contact points made from 5-mil wire were used. The collector was electrically formed by passing large currents in the reverse direction. This reduced the resistance of the collector in the reverse direction, improving the transistor action. However, it remained considerably higher than that of the collector on the oxidized surface.

While there are many ways of plotting the data, we have chosen to give the collector voltage, V_c, as a function of the collector current, I_c, with the emitter current, I_e, taken as a parameter. This plot shows in a direct manner the influence of the emitter current on the current-voltage characteristic of the collector. The curve corresponding to $I_e=0$ is just the normal reverse characteristic of the collector as a rectifier. The other curves show how the characteristic shifts to the right, corresponding to larger collector currents, with increase in emitter current. It may be noted that the change in collector current for fixed collector voltage is larger than the change in emitter current. The current amplification factor, α, defined by

$$\alpha = -(\partial I_c/\partial I_e)_{V_c=\text{const.}}, \qquad \text{(II.1)}$$

is between 2 and 3 throughout most of the plot.

The dotted lines on Fig. 3 correspond to constant values of the emitter voltage, V_e. By interpolating between the contours, all four variables corresponding to a given operating point may be obtained. The V_e contours reach a maximum for I_e about 0.7 ma and have a negative slope beyond. To the left of the maximum, V_e increases with I_e as one follows along a line corresponding to $V_c=$const. To the right, V_e decreases as I_e increases, corresponding to a negative input admittance. For given values of V_e and V_c, there are two possible operating points. Thus for $V_e=0.1$ and $V_c=-20$ one may have $I_e=0.3$ ma, $I_c=-1.1$ ma or $I_e=1.0$, $I_c=-2.7$.

The negative resistance and instability result from the effect of the collector current on the emitter current.[1] The collector current lowers the potential of the surface in the vicinity of the emitter and increases the effective bias on the emitter by an equivalent amount. This potential drop is $R_F I_c$, where R_F is a feed-back resistance which may depend on the currents flowing. The effective bias on the emitter is then $V_e - R_F I_c$, and we may write

$$I_e = f(V_e - R_F I_c), \qquad \text{(II.2)}$$

where the function gives the forward characteristic of the emitter point. In some cases R_F is approximately constant over the operating range; in other cases R_F decreases with increasing I_e as the conductivity of the germanium in the vicinity of the points increases with forward current. Increase of I_e by a change of V_e increases the magnitude of I_c, which by the feedback still further increases I_e. Instability may result. Some consequences will be discussed further in connection with the a.c. characteristics.

Also shown in Fig. 3 is a load line corresponding to a battery voltage of -100 in the output circuit and a load, R_L, of 40,000 ohms, the equation of the line being

$$V_c = -100 - 40\times 10^3 I_c. \qquad \text{(II.3)}$$

The load is an approximate match to the collector resistance, as given by the slope of the solid lines. If operated between the points P_1 and P_2, the output voltage is 8.0 volts r.m.s. and the output current is 0.20 ma. The corresponding values at the input are 0.07 and 0.18, so that the over-all power gain is

$$\text{Gain} \sim 8\times 0.20/(0.07\times 0.18) \sim 125, \qquad \text{(II.4)}$$

which is about 21 db. This is the available gain for a generator with an impedance of 400 ohms, which is an approximate match for the input impedance.

We turn next to the equations for the a.c. characteristics. For small deviations from an operating point, we may write

$$\Delta V_e = R_{11}\Delta I_e + R_{12}\Delta I_c, \qquad \text{(II.5)}$$

$$\Delta V_c = R_{21}\Delta I_e + R_{22}\Delta I_c, \qquad \text{(II.6)}$$

in which we have taken the currents as the independent variables and the directions of currents and voltages as in Fig. 1. The differentials represent small changes from the operating point, and may be small a.c. signals. The coefficients are defined by:

$$R_{11} = (\partial V_e/\partial I_e)_{I_c=\text{const.}}, \qquad \text{(II.7)}$$

$$R_{12} = (\partial V_e/\partial I_c)_{I_e=\text{const.}}, \qquad \text{(II.8)}$$

$$R_{21} = (\partial V_c/\partial I_e)_{I_c=\text{const.}}, \qquad \text{(II.9)}$$

$$R_{22} = (\partial V_c/\partial I_c)_{I_e=\text{const.}}, \qquad \text{(II.10)}$$

These coefficients are all positive and have the dimensions of resistances. They are functions of the d.c. bias currents, I_e and I_c which define the operating point. For $I_e=0.75$ ma and $I_c=-2$ ma the coefficients of the unit of Fig. 3 have the following approximate values:

$$\begin{aligned} R_{11} &= 800 \text{ ohms}, \\ R_{12} &= 300, \\ R_{21} &= 100{,}000, \\ R_{22} &= 40{,}000. \end{aligned} \qquad \text{(II.11)}$$

Equation (II.5) gives the emitter characteristic. The coefficient R_{11} is the input resistance for a fixed

collector current (open circuit for a.c.). To a close approximation, R_{11} is independent of I_c, and is just the forward resistance of the emitter point when a current I_e is flowing. The coefficient R_{12} is the feedback or base resistance, and is equal to R_F as defined by Eq. (II.2) in case R_F is a constant. Both R_{11} and R_{12} are of the order of a few hundred ohms, R_{12} usually being smaller than R_{11}.

Equation (II.6) depends mainly on the collector and on the flow of holes from the emitter to the collector. The ratio R_{21}/R_{22} is just the current amplification factor α as defined by Eq. (II.1). Thus we may write

$$\Delta V_c = R_{22}(\alpha \Delta I_e + \Delta I_c). \qquad \text{(II.12)}$$

The coefficient R_{22} is the collector resistance for fixed emitter current (open circuit for a.c.), and is the order of 10,000–50,000 ohms. Except in the range of large I_e and small I_c, the value of R_{22} is relatively independent of I_e. The factor α generally is small when I_c is small compared with I_e, and increases with I_c, approaching a constant value the order of 1 to 4 when I_c is several times I_e.

The a.c. power gain with the circuit of Fig. 1 depends on the operating point (the d.c. bias currents) and on the load impedance. The positive feedback represented by R_{12} increases the available gain, and it is possible to get very large power gains by operating near a point of instability. In giving the gain under such conditions, the impedance of the input generator should be specified. Alternatively, one can give the gain which would exist with no feedback. The maximum available gain neglecting feedback, obtained when the load R_L is equal to the collector resistance R_{22} and the impedance of the generator is equal to the emitter resistance, R_{11}, is:

$$\text{Gain} = \alpha^2 R_{22}/4R_{11}, \qquad \text{(II.13)}$$

which is the ratio of the collector to the emitter resistance multiplied by $\frac{1}{4}$ the square of the current amplification factor. This gives the a.c. power delivered to the load divided by the a.c. power fed into the transistor. Substituting the values listed above (Eqs. (II.11)) for the unit whose characteristics are shown in Fig. 3 gives a gain of about 80 times (or 19 db) for the operating point P_0. This is to be compared with the gain of 21 db estimated above for operation between P_1 and P_2. The difference of 2 db represents the increase in gain by feedback, which was omitted in Eq. (II.13).

Equations (II.5) and (II.6) may be solved to express the currents as functions of the voltages, giving

$$\Delta I_e = Y_{11}\Delta V_e + Y_{12}\Delta V_c, \qquad \text{(II.14)}$$

$$\Delta I_c = Y_{21}\Delta V_e + Y_{22}\Delta V_c, \qquad \text{(II.15)}$$

where

$$Y_{11} = R_{22}/D, \quad Y_{12} = -R_{12}/D, \quad Y_{21} = -R_{21}/D, \quad Y_{22} = R_{11}/D, \qquad \text{(II.16)}$$

and D is the determinant of the coefficients

$$D = R_{11}R_{22} - R_{12}R_{21}. \qquad \text{(II.17)}$$

The admittances, Y_{11} and Y_{22} are negative if D is negative, and the transistor is then unstable if the terminals are short-circuited for a.c. currents. Stability can be attained if there is sufficient impedance in the input and output circuits exterior to the transistor. Feedback and instability are increased by adding resistance in series with the base electrode. Further discussion of this subject would carry us too far into circuit theory and applications. From the standpoint of transistor design, it is desirable to keep the feed-back resistance, R_{12}, as small as possible.

Variation with Spacing

One of the important parameters affecting the operation of the transistor is the spacing between the point electrodes. Measurements to investigate this effect have been made on a number of germanium surfaces. Tests were made with use of a micromanipulator to adjust the positions of the points. The germanium was generally in the form of a slab from 0.05 to 0.20 cm thick, the lower surface of which was rhodium plated to form a low resistant contact, and the upper plane surface ground and etched, or otherwise treated to give a surface suitable for transistor action. The collector point was usually kept fixed, since it is more critical, and the emitter point moved. Measurements were made with formed collector points. Most of the data have been obtained on surfaces oxidized as described below.

As expected, the emitter current has less and less influence on the collector as the separation, s,[25] is increased. This is shown by a decrease in R_{21}, or α, with s. The effect of the collector current on the emitter, represented by the feed-back resistance R_{12}, also decreases with increase in s. The other coefficients, R_{11} and R_{22}, are but little influenced by spacing. Figures 4, 5, and 6 illustrate the variation of R_{12} and α with the separation. Shown are results for two different collector points A and B on different parts of the same germanium surface.[26] In making the measurements, the bias currents were kept fixed as the spacing was varied. For collector A, $I_e = 1.0$ ma and $I_c = 3.8$ ma; for collector B, $I_e = 1.0$ ma and $I_c = 4.0$ ma. The values of R_{11} and R_{22} were about 300 and 10,000, respectively, in both cases.

[25] Measured between centers of the contact areas.

[26] The surface had been oxidized, and potential probe measurements (reference 2) gave evidence for considerable surface conductivity.

Figure 5 shows that α decreases approximately exponentially with s for separations from 0.005 cm to 0.030 cm, the rate of decrease being about the same in all cases. Extrapolating down to $s=0$ indicates that a further increase of only about 25 percent in α could be obtained by decreasing the spacing below 0.005 cm.

Figure 6 shows that the decrease of α with distance is dependent on the germanium sample used. Curve 1 is similar to the results in Fig. 5. Curve 2 is for a germanium slice with the same surface treatment but from a different melt.

Figure 4 shows the corresponding results for R_{12}. There is an approximate inverse relationship between R_{12} and s.

Another way to illustrate the decreased influence of the emitter on the collector with increase in spacing is to plot the collector characteristic for fixed emitter current at different spacings. Figure 7 is such a plot for a different surface which was ground flat, etched, and then oxidized at 500°C in moist air for 1 hour. The resultant oxide film was washed off.[27] The emitter current, I_e, was kept constant at 1.0 ma.

Data taken on the same surface have been plotted in other ways. As the spacing increases, more emitter current is required to produce the same change in collector current. The fraction of the emitter current which is effective at the collector decreases with spacing. It is of interest to keep V_c and I_c fixed by varying I_e as s is changed and to plot the values of I_e so obtained as a function of s. Such a plot is shown in Fig. 8. The collector voltage, V_c, is fixed at -15 volts. Curves are shown for $I_c=-3$, -4, -6, and -8 ma. We may define a geometrical factor, g, as the ratio of I_e extrapolated to zero spacing to the value of g at the separation s:

$$g(s)=(I_e(0)/I_e(s))_{V_c,\ I_c=\text{const.}}. \qquad \text{(II.18)}$$

It is to be expected that $g(s)$ will depend on I_c, as it is the collector current which provides the field which draws the holes into the collector. For the same reason, it is expected that $g(s)$ will be relatively independent of V_c. This was indeed found to be true in this particular case and values $V_c=-5$, -10, and -15 were used in Fig. 9 which gives a plot of g *versus* s for several values of I_c. The dotted lines give the extrapolation to $s=0$. As expected, g increases with I_c for a fixed s. The different curves can be brought into approximate agreement by taking $s/I_c^{\frac{1}{2}}$ as the independent variable, and this is done in Fig. 10. As will be discussed in Section V, such a relation is to be expected if g depends on the transit time for the holes.

[27] Potential probe measurements on the same surface, given in reference 2, gave evidence of surface conductivity.

Variation with Temperature

Only a limited amount of data has been obtained on the variation of transistor characteristics with temperature.[5] It is known that the reverse characteristic of the germanium diode varies rapidly with temperature, particularly in the case of units with high reverse resistance. In the transistor, the collector is electrically formed in such a way as to have relatively low reverse resistance, and its characteristic is much less dependent on temperature. Both R_{22} and R_{11} decrease with increase in T, R_{22} usually decreasing more rapidly than R_{11}. The feedback resistance, R_{12}, is relatively independent of temperature. The current multiplication factor, α, increases with temperature, but the change is not extremely rapid. Figure 11 gives a plot of α *versus* T for two experimental units. The d.c. bias currents are kept fixed as the temperature is varied. The over-all change in α from -50°C to $+50$°C is only about 50 percent. The increase in α with T results in an increase in power gain with temperature. This may be nullified by a decrease in the ratio R_{22}/R_{11}, so that the over-all gain at fixed bias current may have a negative temperature coefficient.

Variation with Frequency

Equations (II.5) and (II.6) may be used to describe the a.c. characteristics at high frequencies if the coefficients are replaced by general impedances. Thus if we use the small letters i_e, v_e, i_c, v_c, to denote the amplitude and phase of small a.c. signals about a given operating point, we may write

$$v_e=Z_{11}i_e+Z_{12}i_c, \qquad \text{(II.19)}$$

$$v_c=Z_{21}i_e+Z_{22}i_c. \qquad \text{(II.20)}$$

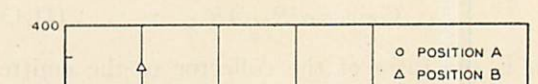

FIG. 4. Dependence of feed-back resistance R_{12} on electrode separation for two different parts A and B, of the same germanium surface. The surface had been oxidized by heating in air.

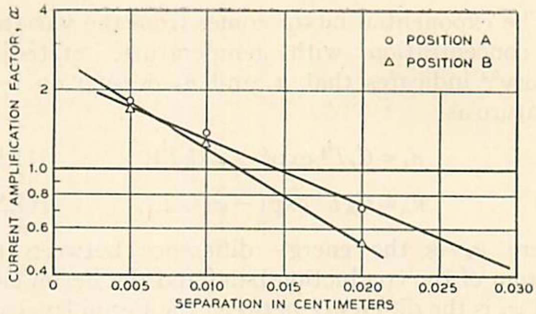

FIG. 5. Dependence of current amplification factor α on electrode separation for formed and unformed collector points. Positions A and B as in Fig. 4.

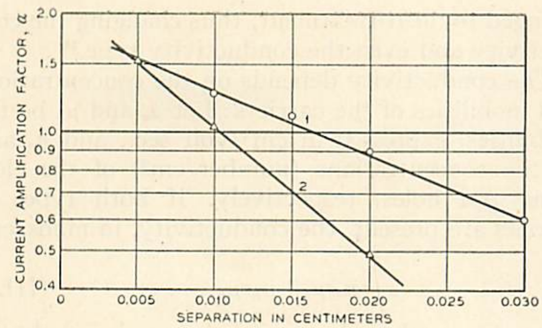

FIG. 6. Dependence of current amplification factor α on electrode separation for germanium surfaces from two different melts, 1 and 2.

Measurements of A. J. Rack and others[28] show that the over-all power gain drops off between 1 and 10 mc/sec. and few units have positive gain above 10 mc/sec. The measurements showed further that the frequency variation is confined almost entirely to Z_{21} or α. The other coefficients, Z_{11}, Z_{12}, and Z_{22} are real and independent of frequency, at least up to 10 mc/sec. Figure 12 gives a plot of α *versus* frequency for an experimental unit. Associated with the drop in amplitude is a phase shift which varies approximately linearly with the frequency. A phase shift in Z_{21} of 90° occurs at a frequency of about 4 mc/sec., corresponding to a delay of about 5×10^{-8} second. Estimates of transit time for the holes to flow from the emitter to the collector, to be made in Section V, are of the same order. These results suggest that the frequency limitation is associated with transit time rather than electrode capacities. Because of the difference in transit times for holes following different paths, there is a drop in amplitude rather than simply a phase shift.

III. ELECTRICAL CONDUCTIVITY OF GERMANIUM

Germanium, like carbon and silicon, is an element of the fourth group of the periodic table, with the same crystal structure as diamond. Each germanium atom has four near neighbors in a tetrahedral configuration with which it forms covalent bonds. The specific gravity is about 5.35 and the melting point 958°C.

The conductivity at room temperature may be either n- or p-type, depending on the nature and concentration of impurities. Scaff, Theuerer, and Schumacher[29] have shown that group III elements with one less valence electron, give p-type conductivity, group V elements, with one more valence electron, give n-type conductivity. This applies to both germanium and silicon. There is evidence that both acceptor (p-type) and donor (n-type) impurities are substitutional.[30]

A schematic energy level diagram[31] which shows the allowed energy levels for the valence electrons in a semiconductor like germanium is given in Fig. 13. There is a continuous band of levels, the filled band, normally occupied by the electrons in the valence bonds, an energy gap, E_G, in which there are no levels of the ideal crystal, and then another continuous band of levels, the conduction band, normally unoccupied. There are just sufficient levels in the filled band to accommodate the four valence electrons per atom. The acceptor impurity levels, which lie just above the filled band, and the donor levels, just below the conduction band, correspond to electrons localized about the impurity atoms. Donors are normally neutral, but become positively charged by excitation of an electron to the conduction band, an energy E_D being acquired. Acceptors, normally neutral, are negatively ionized by excitation of an electron from the filled band, an energy E_A being required. Both E_D and E_A are so small in germanium that practically all donors and acceptors are ionized at room temperature. If only donors are present, the concentration of conduction electrons is equal to the concentration of donors, and the conductivity is n-type. If only acceptors are present, the concentration of missing electrons, or holes, is equal to that of the acceptors, and the conductivity is p-type.

It is possible to have both donor and acceptor type impurities present in the same crystal. In this case, electrons will be transferred from the donor levels to the lower lying acceptor levels. The conductivity type then depends on which is in excess, and the concentration of carriers is equal to the difference between the concentrations of donors and acceptors. It is probable that impurities of both types are present in high back-voltage germanium. The relative numbers in solid solution can be

[28] Unpublished data.

[29] J. H. Scaff, H. C. Theuerer, and E. E. Schumacher, "P-type and N-type Silicon and the Formation of the Photovoltaic Barrier in Silicon" (in publication).

[30] G. L. Pearson and J. Bardeen, Phys. Rev. 75, 865 (1949).

[31] See, for example, reference 6, Chap. 3.

changed by heat treatment, thus changing the conductivity and even the conductivity type.[13]

The conductivity depends on the concentrations and mobilities of the carriers: Let μ_e and μ_h be the mobilities, expressed in cm^2/volt sec., and n_e and n_h the concentrations (number/cm^3) of the electrons and holes, respectively. If both types of carriers are present, the conductivity, in mhos/cm, is

$$\sigma = n_e e \mu_e + n_h e \mu_h, \qquad \text{(III.1)}$$

where e is the electronic charge in coulombs (1.6×10^{-19}).

Except for relatively high concentrations ($\sim 10^{17}$/cm^3 or larger), or at low temperatures, the mobilities in germanium are determined mainly by lattice scattering and so should be approximately the same in different samples. Approximate values, estimated from Hall and resistivity data obtained at Purdue University[32] and at the Bell Telephone Laboratories[33] are:

$$\mu_h = 5 \times 10^6 T^{-\frac{3}{2}}, \qquad \text{(III.2)}$$

$$\mu_e = 6.5 \times 10^6 T^{-\frac{3}{2}} (\text{cm}^2/\text{volt sec.}), \qquad \text{(III.3)}$$

in which T is the absolute temperature. There is a considerable spread among the different measurements, possibly arising from inhomogeneity of the samples. The temperature variation is as indicated by theory. These equations give $\mu_h \sim 1000$ and $\mu_e \sim 1300$ cm^2/volt sec. at room temperature. The resistivity of the germanium used varies from about 1 to 30 ohm cm, corresponding to values of n_e between 1.5×10^{14} and 4×10^{15}/cm^3.

At high temperatures, electrons may be thermally excited from the filled band to the conduction band, an energy E_G being required. Both the excited electron and the hole left behind contribute to the conductivity. The conductivities of all samples approach the same limiting values, regardless of impurity concentration, given by an equation of the form

$$\sigma = \sigma_\infty \exp(-E_G/2kT), \qquad \text{(III.4)}$$

where k is Boltzmann's constant. For germanium, σ_∞ is about 3.3×10^4 mhos/cm and E_G about 0.75 ev.

The exponential factor comes from the variation of concentration with temperature. Statistical theory[34] indicates that n_e and n_h depend on temperature as

$$n_e = C_e T^{\frac{3}{2}} \exp(-\varphi_e/kT), \qquad \text{(III.5a)}$$

$$n_h = C_h T^{\frac{3}{2}} \exp(-\varphi_h/kT), \qquad \text{(III.5b)}$$

where φ_e is the energy difference between the bottom of the conduction band and the Fermi level and φ_h is the difference between the Fermi level and the top of the filled band. The position of the Fermi level depends on the impurity concentration and on temperature. The theory gives

$$C_e \sim C_h \sim 2(2\pi mk/h^2)^{\frac{3}{2}} \sim 5 \times 10^{15}, \qquad \text{(III.6)}$$

where m is an effective mass for the electrons (or holes) and h is Plank's constant. The numerical value is obtained by using the ordinary electron mass for m.

The product $n_e n_h$ is independent of the position of the Fermi level, and thus of impurity concentration, and depends only on the temperature. From Eqs. (III.5a) and (III.5b)

$$n_e n_h = C_e C_h T^3 \exp(-E_G/kT). \qquad \text{(III.7)}$$

In the intrinsic range, we may set $n_e = n_h = n$, and find, using (III.1), (III.2), and (III.3), an expression of the form (III.4) for σ with

$$\sigma_\infty = 11.5 \times 10^6 e (C_e C_h)^{\frac{1}{2}}. \qquad \text{(III.8)}$$

Using the theoretical value (III.6) for $(C_e C_h)^{\frac{1}{2}}$, we find

$$\sigma_\infty = 0.9 \times 10^4 \text{ mhos/cm},$$

as compared with the empirical value of 3.3×10^4, a difference of a factor of 3.6. A similar discrepancy for silicon appears to be related to a variation of E_G with temperature. With an empirical value of

$$C_e C_h = 25 \times 10^{30} \times 3.6^2 \sim 3 \times 10^{32}, \qquad \text{(III.9)}$$

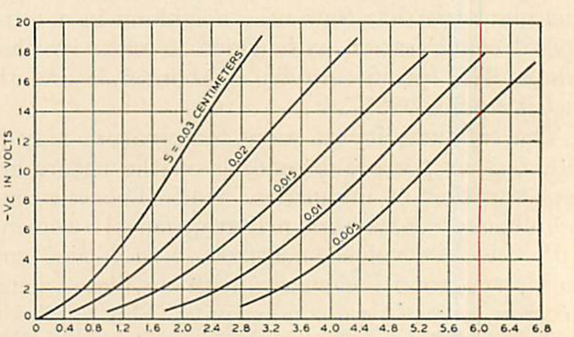

FIG. 7. Collector characteristic V_c *vs.* I_c for fixed I_e but variable distance of separation.

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[32] Lark-Horovitz, Middleton, Miller, and Walerstein, Phys. Rev. 69, 258 (1946).

[33] Hall and resistivity data at the Bell Laboratories were obtained by G. L. Pearson on samples furnished by J. H. Scaff and H. C. Theuerer. *Added in proof:* Recent hall measurements of G. L. Pearson on single crystals of n- and p-type germanium give values of 2700 and 1600 cm^2/volt sec. for electrons and holes, respectively, at room temperature. The latter value has been confirmed by J. R. Haynes by measurements of the drift velocity of holes injected into n-type germanium. These values are higher, particularly for electrons, than earlier measurements on polycrystalline samples. Use of the new values will modify some of the numerical estimates made herein, but the orders of magnitude, which are all that are significant, will not be affected. W. Ringer and H. Welker, Zeits. f. Naturforschung 1, 20 (1948), give a value of 2000 cm^2/volt sec. for high resistivity n-type germanium.

[34] See R. H. Fowler, *Statistical Mechanics* (Cambridge University Press, London, 1936), second edition.

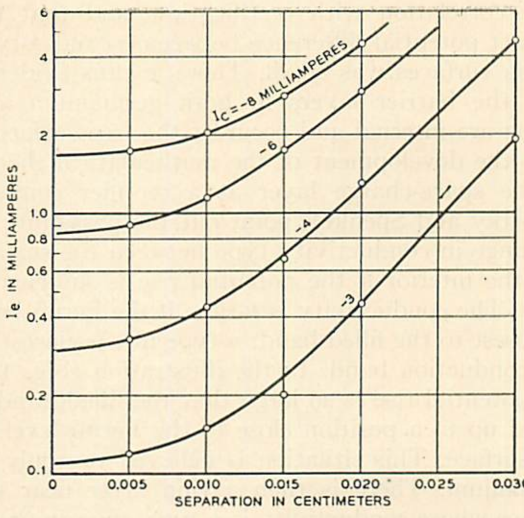

FIG. 8. Emitter current I_e *vs.* separation for fixed I_c and V_c.

Eq. (III.7) gives

$$n_e n_h \sim 10^{27}/\text{cm}^6 \qquad \text{(III.10)}$$

when evaluated for room temperature. Thus for $n_e \sim 10^{15}/\text{cm}^3$, n_h is the order of 10^{12}. The equilibrium concentration of holes is small.

Below the intrinsic temperature range, n_e is approximately constant and n_h varies as

$$n_h = (C_e C_h T^3 / n_e) \exp(-E_G/kT). \qquad \text{(III.11)}$$

IV. THEORY OF THE DIODE CHARACTERISTIC

Characteristics of metal point-germanium contacts include high forward currents, as large as 5 to 10 ma at 1 volt, small reverse currents, corresponding to resistances as high as one megohm or more at reverse voltages up to 30 volts, and the ability to withstand large voltages in the reverse direction without breakdown. A considerable variation of rectifier characteristics is found with changes in preparation and impurity content of the germanium, surface treatment, electrical power or forming treatment of the contacts, and other factors.

A typical d.c. characteristic of a germanium rectifier[35] is illustrated in Fig. 14. The forward voltages are indicated on an expanded scale. The forward current at 1-volt bias is about 3.5 ma and the differential resistance is about 200 ohms. The reverse current at 30 volts is about 0.02 ma and the differential resistance about 5×10^5 ohms. The ratio of the forward to the reverse current at 1-volt bias is about 500. At a reverse voltage of about 160 the differential resistance drops to zero, and with further increase in current the voltage across the unit drops. The nature of this negative resistance portion of the curve is not completely understood, but it is believed to be associated with thermal effects. Successive points along the curve correspond to increasingly higher temperatures of the contact. The peak value of the reverse voltage varies among different units. Values of more than 100 volts are not difficult to obtain.

Theories of rectification as developed by Mott,[36] Schottky,[37] and others[38] have not been successful in explaining the high back-voltage characteristic in a quantitative way. In the following we give an an outline of the theory and its application to germanium. It is believed that the high forward currents can now be explained in terms of a flow of holes. The type of barrier which gives a flow of carriers of conductivity type opposite to that of the base material is discussed. It is possible that a hole current also plays an important role in the reverse direction.

The Space-Charge Layer

According to the Mott-Schottky theory, rectification results from a potential barrier at the

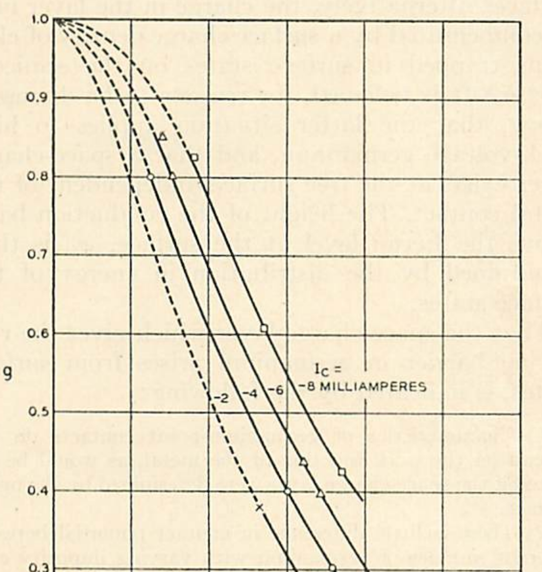

FIG. 9. The factor g is the ratio of the emitter current extrapolated to $s=0$ to that at electrode separation s required to give the same collector current, I_c and voltage, V_c. Plot shows variation of g with s for different I_c. The factor is independent of V_c over the range plotted.

[35] From unpublished data of K. M. Olsen.

[36] N. F. Mott, Proc. Roy. Soc. **171A**, 27 (1939).

[37] W. Schottky, Zeits. f. Physik **113**, 367 (1939); Physik. Zeits. **41**, 570 (1940); Zeits. f. Physik. **118**, 539 (1942). Also see reference 18.

[38] See reference 6, Chap. 4.

contact which impedes the flow of electrons between the metal and the semiconductor. A schematic energy level diagram of the barrier region, drawn roughly to scale for germanium, is given in Fig. 15. There is a rise in the electrostatic potential energy of an electron at the surface relative to the interior which results from a space-charge layer in the semiconductor next to the metal contact. The space charge arises from positively ionized donors, that is from the same impurity centers which give the conduction electrons in the body of the semiconductor. In the interior, the space charge of the donors is neutralized by the space charge of the conduction electrons, which are present in equal numbers. Electrons are drained out of the space-charge layer near the surface, leaving the immobile donor ions.

The space-charge layer may be a result of the metal-semiconductor contact, in which case the positive charge in the layer is compensated by an induced charge of opposite sign on the metal surface. Alternatively, the charge in the layer may be compensated by a surface charge density of electrons trapped in surface states on the semiconductor.[9] It is believed, for reasons to be discussed below, that the latter situation applies to high back-voltage germanium, and that a space-charge layer exists at the free surface, independent of the metal contact. The height of the conduction band above the Fermi level at the surface, φ_s, is then determined by the distribution in energy of the surface states.

That the space-charge layer which gives the rectifying barrier in germanium arises from surface states, is indicated by the following:

(1) Characteristics of germanium-point contacts do not depend on the work function of the metal, as would be expected if the space-charge layer were determined by the metal contact.

(2) There is little difference in contact potential between different samples of germanium with varying impurity concentration. Benzer[39] found less than 0.1-volt difference between samples ranging from n-type with 2.6×10^{18} carriers/cm^3 to p-type with 6.4×10^{18} carriers/cm^3. This is much less than the difference of the order of the energy gap, 0.75 volt, which would exist if there were no surface effects.

(3) Benzer[40] has observed the characteristics of contacts formed from two crystals of germanium. He finds that in both directions the characteristic is similar to the reverse characteristic of one of the crystals in contact with a highly conducting metal-like germanium crystal.

(4) One of the authors[11] has observed a change in contact potential with light similar to that expected for a barrier layer at the free surface.

Prior to Benzer's experiments, Meyerhof[8] had shown that the contact potential difference measured between different metals and silicon showed little correlation with rectification, and that the contact potential difference between n- and p-type silicon surfaces was small. There is thus evidence that the barrier layers in both germanium and silicon are internal and occur at the free surface.[41]

In the development of the mathematical theory of the space-charge layer at a rectifier contact, Schottky and Spenke[18] point out the possibility of a change in conductivity type between the surface and the interior if the potential rise is sufficiently large. The conductivity is p-type if the Fermi level is closest to the filled band, n-type if it is closest to the conduction band. In the illustration (Fig. 15), the potential rise is so large that the filled band is raised up to a position close to the Fermi level at the surface. This situation is believed to apply to germanium. There is then a thin layer near the surface whose conductivity is p-type, superimposed on the n-type conductivity in the interior. Schottky

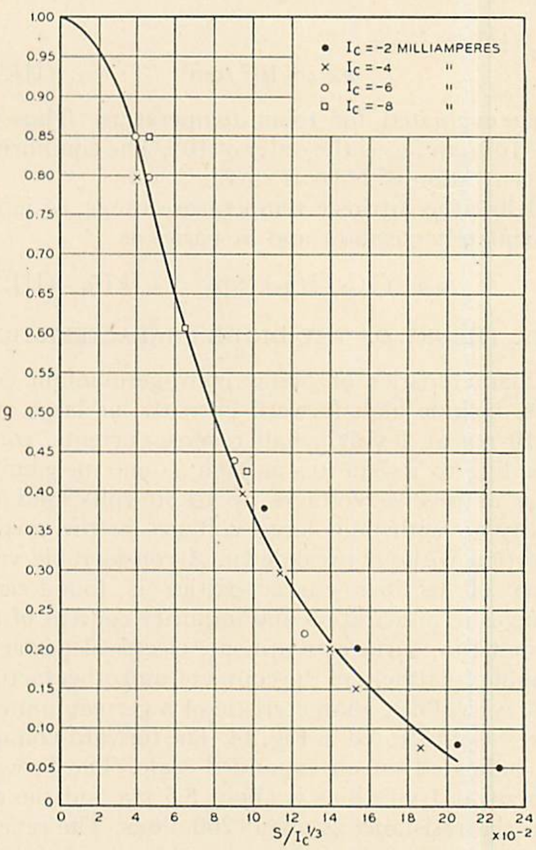

FIG. 10. The factor g (Fig. 9) plotted as a function of $s/I_c^{1/3}$, with s in cm and I_c in amp.

[39] S. Benzer, *Progress Report*, Contract No. W-36-039-SC-32020, Purdue University (Sept. 1–Nov. 30, 1946).

[40] S. Benzer, Phys. Rev. **71**, 141 (1947).

[41] Further evidence that the barrier is internal comes from some unpublished experiments of J. R. Haynes with the transistor. Using a fixed collector point, and keeping a fixed distance between emitter and collector, he varied the material used for the emitter point. He used semiconductors as well as metals for the emitter point. While the impedance of the emitter point varied, it was found that equivalent emitter currents give changes in current at the collector of the same order for all materials used. It is believed that in all cases a large part of the forward current consists of holes.

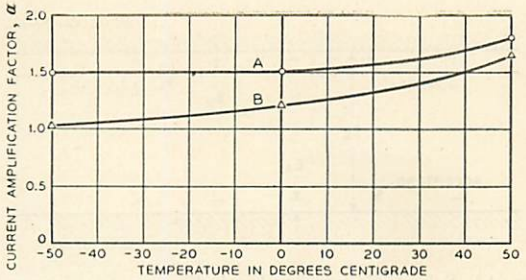

FIG. 11. Current amplification factor α *vs.* temperature for two experimental units A and B.

and Spenke call the layer of opposite conductivity type an inversion region.

Referring to Eqs. (III.5a) and (III.5b) for the concentrations, it can be seen that since C_e and C_h are of the same order of magnitude, the conductivity type depends on whether φ_e is larger or smaller than φ_h. The conductivity is n-type when

$$\varphi_e < \tfrac{1}{2}E_G, \quad \varphi_h > \tfrac{1}{2}E_G, \tag{IV.1}$$

and is p-type when the reverse situation applies. The maximum resistivity occurs at the position where the conductivity type changes and

$$\varphi_e \sim \varphi_h \sim \tfrac{1}{2}E_G. \tag{IV.2}$$

The change from n- to p-type will occur if

$$\varphi_s > \tfrac{1}{2}E_G, \tag{IV.3}$$

or if the over-all potential rise, φ_b, is greater than

$$\tfrac{1}{2}E_G - \varphi_{e0}, \tag{IV.4}$$

where φ_{e0} is the value of φ_e in the interior. Since for high back-voltage germanium, $E_G \sim 0.75$ ev and $\varphi_{e0} \sim 0.25$ ev, a rise of more than 0.12 ev is sufficient for a change of conductivity type to occur. A rise of 0.50 ev will bring the filled band close to the Fermi level at the surface.

Schottky[37] relates the thickness of the space-

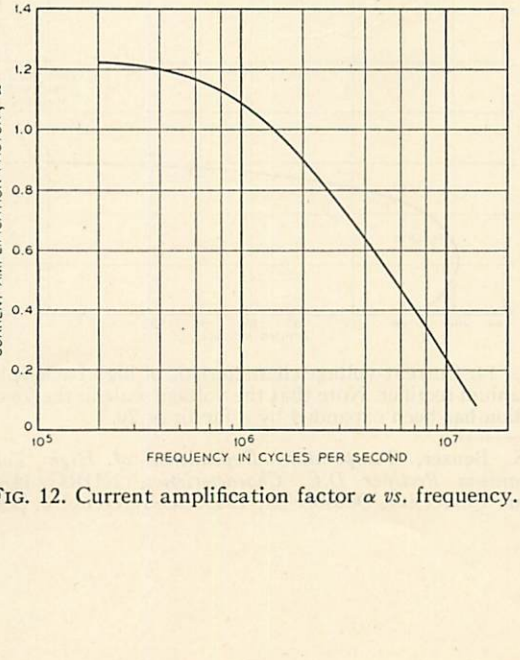

FIG. 12. Current amplification factor α *vs.* frequency.

charge layer with a potential rise as follows. Let ρ be the average change density, assumed constant for simplicity, in the space-charge layer. In the interior ρ is compensated by the space charge of the conduction electrons. Thus, if n_0 is the normal concentration of electrons,[42]

$$\rho = en_0. \tag{IV.5}$$

Integration of the space-charge equations gives a parabolic variation of potential with distance, and the potential rise, φ_b, is given in terms of the thickness of the space-charge layer, l, by the equation

$$\varphi_b = 2\pi e\rho l^2/\kappa = 2\pi e^2 n_0 l^2/\kappa. \tag{IV.6}$$

For

$$\varphi_b = \varphi_s - \varphi_{e0} \sim 0.5 \text{ ev} \sim 8\times 10^{-13} \text{ erg},$$

and

$$n_0 \sim 10^{15}/\text{cm}^3,$$

the barrier thickness, l, is about 10^{-4} cm. The dielectric constant, κ, is about 18 in germanium.

When a voltage V_a is applied to a rectifying contact, there will be a drop V_b across the space-charge layer itself and an additional drop, IR_s, in the body of the germanium which results from the spreading resistance, R_s, so that

$$V_a = V_b + IR_s. \tag{IV.7}$$

The potential energy drop, $-eV_b$, is superimposed on the drop φ_b which exists under equilibrium conditions. For this case Eq. (IV.6) becomes

$$\varphi_b - eV_b = 2\pi e^2 n_0 l^2/\kappa. \tag{IV.8}$$

The potential V_b is positive in the forward direction, negative in the reverse. A reverse voltage increases the thickness of the layer, a forward voltage decreases the thickness of the layer. The barrier disappears when $eV_b = \varphi_b$, and the current is then limited entirely by the spreading resistance in the body of the semiconductor.

The electrostatic field at the contact is

$$F = 4\pi e n_0 l/\kappa = (8\pi n_0(\varphi_b - eV_b)/\kappa)^{\frac{1}{2}}. \tag{IV.9}$$

For $n_0 \sim 10^{15}$, $l \sim 10^{-4}$, and $\kappa \sim 18$, the field F is about 30 e.s.u. or 10,000 volts/cm. The field increases the current flow in much the way the current from a thermionic emitter is enhanced by an external field.

Previous theories of rectification have been based on the flow of only one type of carrier, i.e., electrons in an n-type or holes in a p-type semiconductor. If the barrier layer has an inversion region, it is necessary to consider the flow of both types of carriers. Some of the hitherto puzzling features of the germanium diode characteristic can be explained by the hole current. While a complete

[42] The space charge of the holes in the inversion region of the barrier layer is neglected for simplicity.

theoretical treatment has not been carried out, we will give an outline of the factors involved and then give separate discussions for the reverse and forward directions.

The current of holes may be expected to be important if the concentration of holes at the semiconductor boundary of the space-charge layer is as large as the concentration of electrons at the metal-semiconductor interface. In equilibrium, with no current flow, the former is just the hole concentration in the interior, n_{h0}, which is given by

$$n_{h0}=C_hT^{\frac{3}{2}}\exp(-\varphi_{h0}/kT), \qquad \text{(IV.10)}$$

where φ_{h0} is the energy difference between the Fermi level in the interior and the top of the filled band. The concentration of electrons at the interface is given by:

$$n_{em}=C_eT^{\frac{3}{2}}\exp(-\varphi_s/kT). \qquad \text{(IV.11)}$$

Since C_h and C_e are of the same order, n_{h0} will be larger than n_{em} if φ_s is larger than φ_{h0}. This latter condition is met if the hole concentration at the metal interface is larger than the electron concentration in the interior. The concentrations will, of course, be modified when a current is flowing, but the criterion just given is nevertheless a useful guide. The criterion applies to an inversion barrier layer regardless of whether it is formed by the metal contact or is of the surface states type. In the latter case, as discussed in the Introduction, a lateral flow of holes along the surface layer into the contact may contribute to the current.

Two general theories have been developed for the current in a rectifying junction which apply in different limiting cases. The diffusion theory applies if the current is limited by the resistance of the space-charge layer. This will be the case if the mean free path is small compared with the thickness of the layer, or, more exactly, small compared with the distance required for the potential energy to drop kT below the value at the contact. The diode theory applies if the current is limited by the thermionic emission current over the barrier. In germanium, the mean free path (10^{-5} cm) is of the same order as the barrier thickness. Analysis shows, however, that scattering in the barrier is unimportant and that it is the diode theory which should be used.[43]

Reverse Current

Different parts of the d.c. current-voltage characteristics require separate discussion. We deal first with the reverse direction. The applied voltages are assumed large compared with kT/e (0.025 volt at room temperature), but small compared with the peak reverse voltage, so that thermal effects are

[43] Reference 6, Chapter 4.

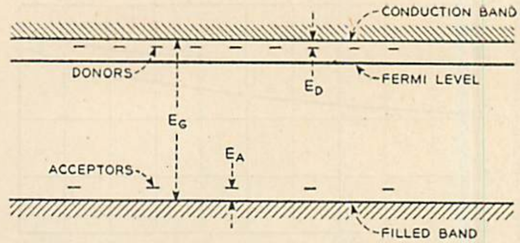

FIG. 13. Schematic energy level diagram for germanium showing filled and conduction bands and donor and acceptor levels.

unimportant. Electrons flow from the metal point contact to the germanium, and holes flow in the opposite direction.

Benzer[44] has made a study of the variation of the reverse characteristic with temperature. He divides the current into three components whose relative magnitudes vary among different crystals and which vary in different ways with temperature. These are:

(1) A saturation current which rises very rapidly with applied voltage, approaching a constant value at a fraction of a volt.

(2) A component which increases linearly with the voltage.

(3) A component which increases more rapidly than linearly with the voltage.

The first two increase rapidly with increasing temperature, while the third component is more or less independent of ambient temperature. It is the saturation current, and perhaps also the linear component, which are to be identified with the theoretical diode current.

The third component is the largest in units with low reverse resistance. It is probable that in these units the barrier is not uniform. The largest part of the current, composed of electrons, flows through patches in which the height of the barrier is small.

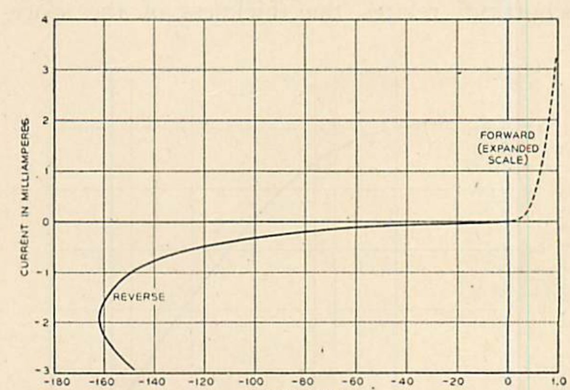

FIG. 14. Current-voltage characteristic of high back-voltage germanium rectifier. Note that the voltage scale in the forward direction has been expanded by a factor of 20.

[44] S. Benzer, *Temperature Dependence of High Voltage Germanium Rectifier D.C. Characteristics*, NDRC 14-579, Purdue University, October 31, 1945. See reference 6, p. 376.

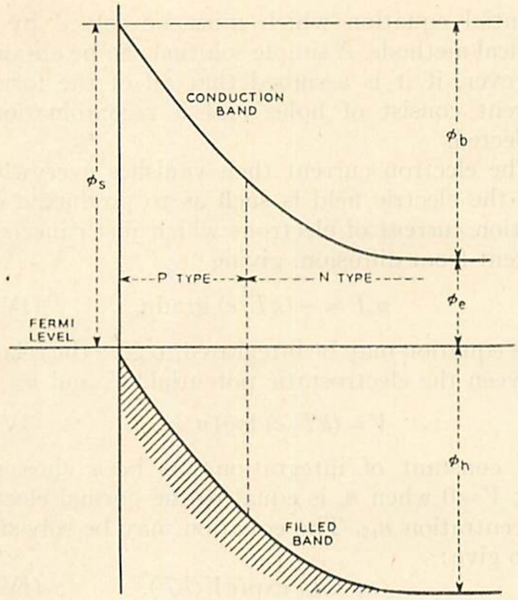

FIG. 15. Schematic energy level diagram of barrier layer at germanium surface showing inversion layer of p-type conductivity.

The electrically formed collector in the transistor may have a barrier of this sort.

Benzer finds that the saturation current predominates in units with high reverse resistance, and that this component varies with temperature as

$$I_s = -I_0 e^{\epsilon/kT}, \qquad \text{(IV.12)}$$

with ϵ nearly 0.7 ev. The negative sign indicates a reverse current. According to the diode theory,[43] one would expect it to vary as

$$I_s = -BT^2 e^{\epsilon/kT}. \qquad \text{(IV.13)}$$

Since ϵ is large, the observed current can be fitted just about as well with the factor T^2 as without. The value of ϵ obtained using (IV. 13) is about 0.6 ev. The saturation current[43] at room temperature varies from 10^{-7} to 10^{-6} amp., which corresponds to values of B in the range of 0.01 to 0.1 amp./deg.^{2}.

The theoretical value of B is 120 times the contact area, A_c. Taking $A_c \sim 10^{-6}$ cm^2 as a typical value for the area of a point contact gives $B \sim 10^{-4}$ amp./deg.^{2} which is only about 1/100 to 1/1000 of the observed. It is difficult to reconcile the magnitude of the observed current with the large temperature coefficient, and it is possible that an important part of the total flow is a current of holes into the contact. Such a current particularly is to be expected on surfaces which exhibit an appreciable surface conductivity.

Neglecting surface effects for the moment, an estimate of the saturation hole current might be obtained as follows. The number of holes entering the space-charge region per second is[45]

$$n_{hb} v_a A_c / 4,$$

where n_{hb} is the hole concentration at the semiconductor boundary of the space-charge layer and v_a is an average thermal velocity ($\sim 10^7$ cm/sec.). The hole current, I_h, is obtained by multiplying by the electronic charge, giving

$$I_h = -n_{hb} e v_a A_c / 4, \qquad \text{(IV.14)}$$

If we set n_{hb} equal to the equilibrium value for the interior, say 10^{12}/cm^3, we get a current $I_h \sim 4 \times 10^{-7}$ amp., which is of the observed order of magnitude of the saturation current at room temperature. With this interpretation, the temperature variation of I_s is attributed to that of n_h, which, according to Eq. (III.11) varies as $\exp(-E_G/kT)$. The observed value of ϵ is indeed almost equal to the energy gap.

The difficulty with this picture is to see how n_{hb} can be as large as n_{h0} when a current is flowing. Holes must move toward the contact area primarily by diffusion, and the hole current will be limited by a diffusion gradient. The saturation current depends on how rapidly holes are generated, and reasonable estimates based on the mean lifetime, τ, yield currents which are several orders of magnitude too small. A diffusion velocity, v_D, of the order

$$v_D \sim (D/\tau)^{\frac{1}{2}}, \qquad \text{(IV.15)}$$

replaces $v_a/4$ is Eq. (IV.14). Setting $D \sim 25$ cm^2/sec. and $\tau \sim 10^{-6}$ sec. gives $v_D \sim 5 \times 10^3$, which would give a current much smaller than the observed. What is needed, then, is some other mechanism which will help maintain the equilibrium concentration near the barrier. Surface effects may be important in this regard.

Forward Current

The forward characteristic is much less dependent on such factors as surface treatment than the reverse. In the range from 0 to 0.4 volt in the forward direction, the current can be fitted quite closely by a semi-empirical expression[46] of the form:

$$I = I_0 (e^{\beta V_b} - 1), \qquad \text{(IV.16)}$$

where V_b is the drop across the barrier resulting from the applied voltage, as defined by Eq. (IV.7). Equation (IV.16) is of the general form to be expected from theory, but the measured value of β is generally less than the theoretical value e/kT (40 volts^{-1} at room temperature). Observed values of β may be as low as 10, and in other units are nearly as high as the theoretical value of 40. The factor I_0 also varies among different units and is of

[45] See, for example, E. H. Kennard, *Kinetic Theory of Gases* (McGraw-Hill Book Company, Inc., New York, 1938), p. 63.
[46] Reference 6, p. 377.

the order 10^{-7} to 10^{-6} ampere. While both experiment and theory indicate that the forward current at large forward voltages is largely composed of holes, the composition of the current at very small forward voltages is uncertain. Small areas of low φ_s, unimportant at large forward voltages, may give most of the current at very small voltages. Currents flowing in these areas will consist largely of electrons.

Above about 0.5 volt in the forward direction, most of the drop occurs across the spreading resistance, R_s, rather than across the barrier. The theoretical expression for R_s for a circular contact of diameter d on the surface of a block of uniform resistivity ρ is:

$$R_s = \rho/2d. \qquad \text{(IV.17)}$$

Taking as typical values for a point contact on high back-voltage germanium, $\rho = 10$ ohm cm and $d = 0.0025$ cm, we obtain $R_s = 2000$ ohms, which is the order of ten times the observed.

As discussed in the Introduction, Bray and others[21, 22] have attempted to account for this discrepancy by assuming that the resistivity decreases with increasing field, and Bray has made tests to observe such an effect. The authors have investigated the nature of the forward current by making potential probe measurements in the vicinity of a point contact.[2] These measurements indicate that there may be two components involved in the excess conductivity. Some surfaces, prepared by oxidation at high temperatures, give evidence for excess conductivity in the vicinity of the point in the reverse as well as in the forward direction. This ohmic component has been attributed to a thin p-type layer on the surface. All surfaces investigated exhibit an excess conductivity in the forward direction which increases with increasing forward current. This second component is attributed to an increase in the concentration of carriers, holes and electrons, in the vicinity of the point with increase in forward current. Holes flow from the point into the germanium and their space charge is compensated by electrons.

The ohmic component is small; if it exists at all, on surfaces treated in the normal way for high back-voltage rectifiers (i.e., ground and etched). The nature of the second component on such surfaces has been shown by more recent work of Shockley, Haynes,[20] and Ryder[23] who have investigated the flow of holes under the influence of electric fields. These measurements prove that the forward current consists at least in large part of holes flowing into the germanium from the contact.

It is of interest to consider the way the concentrations of holes and electrons vary in the vicinity of the point. An exact calculation, including the effect of recombination, leads to a non-linear differential equation which must be solved by numerical methods. A simple solution can be obtained, however, if it is assumed that all of the forward current consist of holes and if recombination is neglected.

The electron current then vanishes everywhere, and the electric field is such as to produce a conduction current of electrons which just cancels the current from diffusion, giving

$$n_e F = -(kT/e)\,\mathrm{grad}\,n_e. \qquad \text{(IV.18)}$$

This equation may be integrated to give the relation between the electrostatic potential, V, and n_e,

$$V = (kT/e)\log(n_e/n_{e0}). \qquad \text{(IV.19)}$$

The constant of integration has been chosen so that $V = 0$ when n_e is equal to the normal electron concentration n_{e0}. The equation may be solved for n_e to give:

$$n_e = n_{e0}\exp(eV/kT). \qquad \text{(IV.20)}$$

If trapping is neglected, electrical neutrality requires that

$$n_e = n_h + n_{e0}. \qquad \text{(IV.21)}$$

Using this relation, and taking n_{e0} a constant, we can express the field F in terms of n_h

$$F = -(kT/e(n_h + n_{e0}))\,\mathrm{grad}\,n_h. \qquad \text{(IV.22)}$$

The hole current density, i_h, is the sum of a conduction current resulting from the field F and a diffusion current:

$$i_h = n_h e \mu_h F - kT\mu_h\,\mathrm{grad}\,n_h. \qquad \text{(IV.23)}$$

Using Eq. (IV.22), we may write this in the form

$$i_h = -kT\mu_h((2n_h + n_{e0})/(n_h + n_{e0}))\,\mathrm{grad}\,n_h. \qquad \text{(IV.24)}$$

The current density can be written

$$i_h = -\mathrm{grad}\,\psi, \qquad \text{(IV.25)}$$

where

$$\psi = kT\mu_h(2n_h - n_{e0}\log((n_h + n_{e0})/n_{e0})). \qquad \text{(IV.26)}$$

Since i_h satisfies a conservation equation,

$$\mathrm{div}\,i_h = 0, \qquad \text{(IV.27)}$$

ψ satisfies Laplace's equation.

If surface effects are neglected and it is assumed that holes flow radially in all directions from the point contact, ψ may be expressed simply in terms of the total hole current, I_h, flowing from the contact:

$$\psi = -I_h/2\pi r. \qquad \text{(IV.28)}$$

Using (IV.26), we may obtain the variation of n_h with r. We are interested in the limiting case in which n_h is large compared with the normal electron concentration, n_{e0}. The logarithmic term in (IV.26) can then be neglected, and we have

$$n_h = I_h/4\pi r \mu_h kT. \qquad \text{(IV.29)}$$

For example, if $I_h = 10^{-3}$ amp., $\mu_h = 10^3$ cm^2/volt sec., and $kT/e = 0.025$ volt, we get, approximately,

$$n_h = 2\times 10^{13}/r. \qquad \text{(IV.30)}$$

For $r \sim 0.0005$ cm, the approximate radius of a point contact,

$$n_h \sim 4\times 10^{16}/\text{cm}^3, \qquad \text{(IV.31)}$$

which is about 40 times the normal electron concentration in high back-voltage germanium. Thus the assumption that n_h is large compared with n_{e0} is valid, and remains valid up to a distance of the order of 0.005 cm, the approximate distance the points are separated in the transistor.

To the same approximation, the field is

$$F = kT/er, \qquad \text{(IV.32)}$$

independent of the magnitude of I_h.

The voltage drop outside of the space-charge region can be obtained by setting n_e in (IV.19) equal to the value at the semiconductor boundary of the space-charge layer. This result holds generally, and does not depend on the particular geometry we have assumed. It depends only on the assumption that the electron current i_e is everywhere zero. Actually i_h will decrease and i_e increase by recombination, and there will be an additional spreading resistance for the electron current.

If it is assumed that the concentration of holes at the metal-semiconductor interface is independent of applied voltage and that the resistive drop in the barrier layer itself is negligible, that part of the applied voltage which appears across the barrier layer itself is:

$$V_b = (kT/e)\log(n_{hb}/n_{h0}), \qquad \text{(IV.33)}$$

where n_{hb} is the hole concentration at the semiconductor boundary of the space-charge layer and n_{h0} is the normal concentration. For $n_{hb} \sim 5\times 10^{16}$ and $n_{h0} \sim 10^{12}$, V_b is about 0.30 volt.

The increased conductivity caused by hole emission accounts not only for the large forward currents, but also for the relatively small dependence of spreading resistance on contact area. At a small distance from the contact, the concentrations and voltages are independent of contact area. The voltage drop within this small distance is a small part of the total and does not vary rapidly with current.

We have assumed that the electron current, I_e, at the contact is negligible compared with the hole current, I_h. An estimate of the electron current can be obtained as follows. From the diode theory,

$$I_e = (en_{eb}v_a A_c/4)\exp(-(\varphi_b - eV_b)/kT), \qquad \text{(IV.34)}$$

since the electron concentration at the semiconductor boundary of the space-charge layer is n_{eb} and the height of the barrier with the voltage applied is $\varphi_b - eV_b$. For simplicity we assume that both n_{eb} and n_{hb} are large compared with n_{e0} so that we may replace n_{eb} by n_{hb} without appreciable error. The latter can be obtained from the value of ψ at the contact:

$$\psi = I_h/4a. \qquad \text{(IV.35)}$$

Expressing ψ in terms of n_{hb}, we find

$$n_{hb} = I_h/8kT\mu_h a. \qquad \text{(IV.36)}$$

Using (IV.33) for V_b, and (III.5b) for n_{h0} we find after some reduction,

$$I_e = I_h^2/I_{\text{crit}}, \qquad \text{(IV.37)}$$

where

$$I_{\text{crit}} = \frac{256 C_h (kT\mu_h)^2 T^{\frac{3}{2}}}{\pi e v_a}\exp(-\varphi_{hm}/kT). \qquad \text{(IV.38)}$$

The energy difference φ_{hm} is the difference between the Fermi level and the filled band at the metal-semiconductor interface. Evaluated for germanium at room temperature, (IV.38) gives

$$I_{\text{crit}} = 0.07\exp(-\varphi_{hm}/kT)\ \text{amp}. \qquad \text{(IV.39)}$$

which is a fairly large current if φ_{hm} is not too large compared with kT. If I_h is small compared with I_{crit}, the electron current will be negligible.

V. THEORETICAL CONSIDERATIONS ON TRANSISTOR ACTION

In this section we discuss some of the problems connected with transistor action, such as:

(1) fields produced by the collector current,
(2) transit times for the holes to flow from emitter to collector,
(3) current multiplication in collector,
(4) feed-back resistance.

We do no more than estimate orders of magnitude. An exact calculation, taking into account the change of conductivity introduced by the emitter current, loss of holes by recombination, and effect of surface conductivity is difficult and is not attempted.

To estimate the field produced by the collector, we assume that the collector current is composed mainly of conduction electrons, and that the electrons flow radially away from the collector. This assumption should be most nearly valid when the collector current is large compared with the emitter current. The field at a distance r from the collector is,

$$F = \rho I_c/2\pi r^2. \qquad \text{(V.1)}$$

For example, if, $\rho = 10$ ohm cm, $I_c = 0.001$ amp., and $r = 0.005$ cm, F is about 100 volts/cm.

The drift velocity of a hole in the field F is $\mu_h F$. The transit time is

$$T = \int \frac{dr}{\mu_h F} = \frac{2\pi}{\mu_h \rho I_c}\int_0^s r^2 dr, \qquad \text{(V.2)}$$

where s is the separation between the emitter and collector. Integration gives,

$$T = 2\pi s^3/3\mu_{h0}I_c. \qquad (V.3)$$

For $s=0.005$ cm, $\mu_h=1000$ cm^2/volt sec., $\rho=10$ ohm cm, and $I_c=0.001$ amp. T is about 0.25×10^{-7} sec. This is of the order of magnitude of the transit times estimated from the phase shift in α or Z_{21}.

The hole current, I_h, is attenuated by recombination in going from the emitter to the collector. If τ is the average lifetime of a hole, I_h will be decreased by a factor, $e^{-T/\tau}$. In Section II it was found that the geometrical factor, g, which gives the influence of separation on the interaction between emitter and collector depends on the variable $s/I_c^{\frac{1}{3}}$. This suggests that the transit time is the most important factor in determining g. An estimate[47] of τ, obtained from the data of Fig. 10, is 2×10^{-7} sec.

Because of the effect of holes in increasing the conductivity of the germanium in the vicinity of the emitter and collector, it can be expected that the field, the lifetime, and the geometrical factor will depend on the emitter current. The effective value of ρ to be used in Eqs. (V.1) and (V.2) will decrease with increase in emitter current. This effect is apparently not serious with the surface used in obtaining the data for Figs. 8 to 10.

Next to be considered is the effect of the space charge of the holes on the barrier layer of the collector. An estimate of the hole concentration can be obtained as follows. The field in the barrier layer is of the order of 10^4 volts/cm. Multiplying by the mobility gives a drift velocity, v_d of 10^7 cm/sec., which is approximately thermal velocity.[48] The hole current is

$$I_h = n_h e v_d A_c, \qquad (V.4)$$

where A_c is the area of the collector contact, and n_h the concentration of holes in the barrier. Solving for the latter, we get

$$n_h = I_h/ev_dA_c. \qquad (V.5)$$

For $I_h=0.001$ amp., $v_d=10^7$ cm/sec. and $A_c=10^{-6}$ cm^2, n_h is about 0.6×10^{15}, which is of the same order as the concentration of donors. Thus the hole current can be expected to alter the space charge in the barrier by a significant amount, and correspondingly alter the flow of electrons from the collector. It is believed that current multiplication (values of $\alpha>1$) can be accounted for along these lines.

As discussed in Section II, there is an influence of collector current on emitter current of the nature of a positive feedback. The collector current lowers the potential of the surface in the vicinity of the emitter by an amount

$$V = \rho I_c/2\pi s. \qquad (V.6)$$

The feed-back resistance R_F as used in Eq. (II.2) is

$$R_F = \rho/2\pi s. \qquad (V.7)$$

For $\rho=10$ ohm cm and $s=0.005$ cm, the value of R_F is about 300 ohms, which is of the observed order of magnitude. It may be expected that R_F will decrease as ρ decreases with increase in emitter current.

The calculations made in this section confirm the general picture which has been given of the way the transistor operates.

VI. CONCLUSIONS

Our discussion has been confined to the transistor in which two point contacts are placed in close proximity on one face of a germanium block. It is apparent that the principles can be applied to other geometrical designs and to other semiconductors. Some preliminary work has shown that transistor action can be obtained with silicon and undoubtedly other semiconductors can be used.

Since the initial discovery, many groups in the Bell Laboratories have contributed to the progress that has been made. This work includes investigation of the physical phenomena involved and the properties of the materials used, transistor design, and measurements of characteristics and circuit applications. A number of transistors have been made for experimental use in a pilot production. Obviously no attempt has been made to describe all of this work, some of which has been reported on in other publications.[5, 19, 20, 23]

In a device as new as the transistor, various problems remain to be solved. A reduction in noise and an increase in the frequency limit are desirable. While much progress has been made toward making units with reproducible characteristics, further improvement in this regard is also desirable.

It is apparent from reading this article that we have received a large amount of aid and assistance from other members of the Laboratories staff, for which we are grateful. We particularly wish to acknowledge our debt to Ralph Bown, Director of Research, who has given us a great deal of encouragement and aid from the inception of the work and to William Shockley, who has made numerous suggestions which have aided in clarifying the phenomena involved.

[47] Obtained by plotting log$g$ *versus* S^3/I_c. This plot is not a straight line, but has an upward curvature corresponding to an increase in τ with separation. The value given is a rough average, corresponding to S^3/I_c the order of 10^{-3} cm^3/amp.

[48] One may expect that the mobility will depend on field strength when the drift velocity is as large as or is larger than thermal velocity. Since ours is a borderline case, the calculation using the low field mobility should be correct at least as to order of magnitude.

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